TABLE III

Experimental Conditions	Time of observation, hours	Number of fission events	Number of fissions gram-hour	Half life
Laboratory attic	381	56	0.013	$1.5 \cdot 10^{19}$ yrs
Cellar (10 meters of soil above apparatus	392	8	0.002	10^{20} yrs

Both the dependence on external conditions (attic, cellar) and the absolute magnitude of the effect (0.002 - 0.013 fissions per gram-hour at sea level) agree well with the results of our experiments on cosmic ray induced fission in heavy nuclei at sea level and at high altitudes. As can be seen from Table III, the effect of fission in the chamber decreases by a factor of 6-7 when the apparatus is moved from attic to cellar. Hence the number of fission events in thorium which can be ascribed to the spontaneous mechanism is not more than 0.002 fission per gram-hour. It is well to note also that at least half of the effect in the cellar must be attributed to the spontaneous fission of uranium nuclei, which constituted 0.006% of the thorium sample investigated (see above).

In this way, our experiments with a big ionization chamber detecting fission fragments indicate that the probability of spontaneous fission in thorium is very small, the half life being more than 10^{20} years.

Our results on thorium fission at sea level differ from those of Segre, which were apparently carried out under the same conditions, by a factor of 10, approximately. If we discard the possibility of trivial mistakes in the work of Segrè, such as the contamination of his chamber by traces of artificial transuranium elements (for instance, Pu^{240}), or the improbable presence in his apparatus of vanishingly small quantities of natural transuranium elements, then considering our data it appears difficult to explain the fact that the rate of spontaneous fission in thorium observed by Segre was a whole order of magnitude larger than the rate induced in thorium by cosmic rays at sea level.

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The Momentum Distribution in the Statistical Model of the Atom

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1. In the statistical model of Thomas-Fermi, the distribution of the angular momentum over the particles can be determined. The number of particles with given angular momentum L is expressed by the equation[1]

$$n(L) = \frac{4L}{\pi} \int [r^2 P^2(r) \quad L^2]^{1/2} \frac{dr}{r^3}, \qquad (1)$$

where $P(r)$ is the maximum momentum for a given point. The mean value of the square of the orbital momentum, $\overline{L^2}$, has been determined by Jensen and Luttinger[2] from the momentum distribution (1). Taking the potential distribution of the atom to be that of the simple Thomas-Fermi model, the latter authors found $\overline{L^2}$, and compared it with the experimental data obtained for the electron level

scheme. Interest exists in a comparison of $\overline{L^2}$ from the Thomas-Fermi model, with exchange interaction of the electrons, with the $\overline{L^2}$ from the simple Thomas-Fermi model, and with experiment.

The mean square of the orbital momentum is given by

$$\overline{L^2} = \frac{1}{Z}\int L^2 n(L)\, dL = \frac{8}{15\pi Z}\int_0^\infty [rP(r)]^5 \frac{dr}{r}. \qquad (2)$$

The value of $\overline{L^2}$ can be determined here if the spatial distribution of particles, associated with $P(r)$, is known. In the case of the atom, $\rho(r)$ or $P(r)$ can be expressed by a self-consistent potential which is obtained as the solution of the Thomas-Fermi equation or of the Thomas-Fermi-Dirac equation. In the latter case, when [1]

$$\rho(r) = \frac{Z}{4\pi\mu^3}\left[\left(\frac{\psi}{x}\right)^{1/2} + \beta_0\right]^3; \quad r = \mu x; \qquad (3)$$

$$\mu = \left(\frac{9\pi^2}{128Z}\right)^{1/3} a_0; \quad \beta_0 = \left(\frac{3}{32\pi^2}\right)^{1/3} Z^{-2/3}$$

where $\psi(x)$ is the solution of the Thomas-Fermi-Dirac equation for the corresponding boundary conditions, we can get the following form for $\overline{L^2}$

$$\overline{L^2} = \frac{2}{5}\left(\frac{3\pi}{4}\right)^{2/3} Z^{7/3}\int_0^\infty x^4\left\{\left(\frac{\psi}{x}\right)^{1/2} + \beta_0\right\}^5 dx. \qquad (4)$$

For the simple Thomas-Fermi model, we get, for $\beta_0 = 0$ ($\psi \to \infty$), [2]

$$\overline{L^2}_{\text{T}-\Phi} = \frac{2}{5}\left(\frac{3\pi}{4}\right)^{2/3} Z^{7/3}\int_0^\infty (x\varphi)^{5/2}\frac{dx}{x} = 0{,}262\cdot Z^{7/3} \qquad (5)$$

(Here the integral is a constant, equal to 0.370.) It is easy to show that the integral in Eq. (4) will also be a function of Z. Making use of the numerical solution of the Thomas-Fermi-Dirac equation for inert gases [3], we found $\overline{L^2}$ for them. These results were plotted on a graph (see the Figure), where the empirical data, determined from the electron level scheme in the atom according to the formula

$$\overline{L^2}_{\text{опыт}} = \frac{1}{Z}\sum_i l_i(l_i+1) \quad (\hbar = 1), \qquad (6)$$

(summed over all Z electrons), are also plotted.

The values of $\overline{L^2}$ obtained from Eq. (4) are in better agreement with experiment at high values of Z than the values of $L^2_{\text{T-F}}$ from Eq. (5). The agreement is somewhat worse for the light atoms. We meet with a similar situation in the calculation of the total binding energy of the atoms, and also in the calculation of the value of Z at which the l state first appears (the "first appearance of the l state", see below). Nevertheless, inclusion of exchange improves the results of the calculations of a whole series of properties of the heavy atoms.

2. The distribution of angular momentum (1) was applied to the calculation of the first appearance in atoms of electrons with given orbital momentum by Fermi, and to the calculation of the limits of the l state of nucleons in nuclei by Ivanenko and Rodichev [4], and by other authors [5-7]*.

There are a number of unclear aspects to this problem. These are connected essentially with the continuous spectrum of the angular momentum in the Thomas-Fermi model. The first appearance of a state with given orbital momentum L is determined by the condition

$$n(L) = 0, \qquad (7)$$

where L is determined to be $l + \frac{1}{2}$ or** $[l(l+1)]^{1/2}$.

As was noted by Paneth [8], the total number of particles in the system for which $n(l) = 1$, is larger by one than the total number of particles for which $n(l) = 0$. The reason for this difference lies in the fact that the momentum is not quantized, so that, when a single particle is added to a given system, it does not receive a definite momentum, exceeding that corresponding to the maximum for the degree of filling. Rather, the particle is "distributed" continuously over the surface of the momentum sphere. Thus the $(N+1)$st particle will be distributed continuously over states with L ranging from zero to $rP_{N+1}(r)$. Therefore, for example, in the simple Thomas-Fermi model, the appearance of the d state in the atom will be found at $Z = 21.0$ from the relation

$$n(L) = 1, \qquad (8)$$

and at $Z = 19.4$ from Eq. (7). The location of the first appearance of the l state can also be determined from the condition that the number of particles with angular momentum greater than a given L,

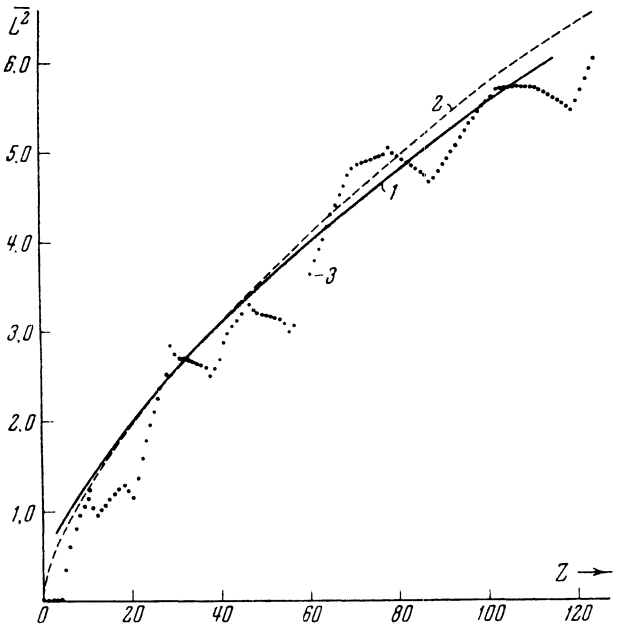
$$n(L) = \int_L^\infty n(L)\, dL$$

will be zero:

$$N(L) = 0, \tag{9}$$

or unity:

$$N(L) = 1. \tag{10}$$



Mean value of the square of the orbital momentum of electrons in an atom. *1.* Thomas-Fermi-Dirac model [from Eq. (4)]; *2.* Thomas-Fermi model [from Eq.(5)]; *3.* empirical values from Eq. (6).

The next larger integer should be used for this first appearance in the results from Eq. (9) and also from Eq. (7). Jensen and Luttinger[2] suggested the determination of the first appearance of l from the relation

$$N\left\{\frac{[l(l+1)]^{1/2}+[l(l-1)]^{1/2}}{2}\right\} - N\left\{\frac{[l(l+1)]^{1/2}+[(l+1)(l+2)]^{1/2}}{2}\right\} = 1 \tag{11}$$

(Here $L = [l(l+1)]^{1/2}$.) We note that the latter relation, in view of the remark of Paneth[8] previously cited, reduces essentially to

$$N\left\{\frac{[l(l+1)]^{1/2}+[l(l-1)]^{1/2}}{2}\right\} = 1.$$

The question naturally arises: which of the determinations (7) - (11) of the first appearance is more accurate? We first note that the determinations from Eqs. (7) and (9) agree with each other. Following the remark of Paneth, we consider those determinations to be more accurate because the particle, as an entity, ought to receive a definite, higher angular momentum, in accordance with quantum mechanics. We should strictly apply the Thomas-Fermi method in this case only for systems with filled shells with spherically symmetric (5).

The determination of the limits of the l states from Eqs. (7) and (9) agrees, in the simple Thomas-Fermi model, with the determination of the first appearance from the condition of the tangency of the abscissa to the curve of "effective potential energy". In the calculation of the exchange interaction, it is impossible to express the entire

potential energy by a potential function and the determination becomes incorrect. The limits of the s, p, d, . . . , states can then be determined from Eq. (7) or Eq. (9), which reduce to the condition

$$[r^3\rho(r)]_{\text{макс}} = (1/24\pi^2)[4l(l+1)]^{3/2}.$$

Substituting $\rho(r)$ from the Thomas-Fermi-Dirac model (3), we get

$$Z_l = \gamma(Z)[4l(l+1)]^{3/2},\quad \gamma(Z) = \frac{1}{6\pi}[(x\psi)^{1/2} + \beta_0 x]^{-3}_{\text{макс}}. \tag{12}$$

By a numerical method, analogous to that of Ivanenko and Larin[9], we found for the limits of the s, p, d, and f states, $Z_l = \frac{1}{2}$, 4, 19, 53, respectively, if $L = [l(l+1)]^{1/2}$, and $Z = 1$, 4, 20, 55 if $L = l + \frac{1}{2}$. Thus the Thomas-Fermi-Dirac model gives $\gamma(Z)$ with $\gamma = 0.155$ (which follows from the simple Thomas-Fermi model) only for sufficiently large Z.

In conclusion, gratitude is expressed to Prof. D. D. Ivanenko and N. N. Kolesnikov for their consideration of the problems examined here.

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* In Born and Yang[7] the parameters of the density distribution of nucleons in the nucleus are determined, in essence, by the number of the "first appearance". In this case there correspond to the numbers of the first appearance of the p, d and f states in reference 7, l of one integer less, i.e., l 0, 1 and 2, respectively. The numbers of the first appearance of the g, h and i states under the same conditions do not agree with experiment.

** The relative difference in the expressions $l + \frac{1}{2}$ and $[l(l+1)]^{1/2}$ is substantially greater for small l, since $l(l1) = (l+\frac{1}{2})^2 - 1/4$.

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The Theory of the Dipole Lattice of Onsager

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IN his paper[1] on phase transitions of the second kind in a plane dipole lattice, Onsager obtained the following expression for the logarithm of the partition function, per particle:

$$\ln\lambda^{(2)}(T) = \frac{1}{2\pi^2}\int_0^{\pi}\int_0^{\pi}\ln(\cosh 2\theta_1\cosh 2\theta_2 - \sinh 2\theta_1\cos\omega_1 - \sinh 2\theta_2\cos\omega_2)\,d\omega_1\,d\omega_2. \tag{1}$$

Here $\theta_n = J_n/kT$ $(n = 1, 2)$; J_n is a constant characterizing the interaction between neighboring dipoles, and T is the temperature. Analysis[1,2] shows that Eq. (1) leads to a logarithmic divergence in the second derivative with respect to temperature, determined by the equation

$$\cosh 2\theta_1\cosh 2\theta_2 - \sinh 2\theta_1 - \sinh 2\theta_2 = 0. \tag{2}$$

Taking one of the interaction constants to be zero, J_2, for example, Eq. (1) becomes a one dimensional integral:

$$\ln\lambda^{(1)}(T) = \frac{1}{2\pi}\int_0^{\pi}\ln(\cosh 2\theta - \sinh 2\theta\cos\omega)\,d\omega = \ln\cosh\theta, \tag{3}$$

which corresponds to a linear chain of dipoles.

It would seem natural[3] to conjecture, that for a three dimensional dipole lattice, $\ln\lambda^{(3)}(T)$ would become a triple integral: