the following approximation was used:

$$W_k\,(E', E)\,dE = gdE\,/\,EE'.$$

Here $W_k(E', E)$ is the probability that a photon of energy E', in its passage through a layer of unit thickness in the cascade, experiences Compton scattering by the electron, after which it will have the energy E; g is a certain constant, equal, for air, to 1.32 MeV; for carbon, 1.53 MeV; for aluminum, 0.839 MeV; for iron, 18.4 MeV; for copper, 0.404 MeV; and for lead 0.175 MeV.

In the work reported in reference 2, a more exact expression was obtained for the "equilibrium" spectrum of photons $\Theta(E)$. Precision in this case is connected with the fact, that for the probability of the Compton-effect the following more exact expression was taken:

$$W_k\,(E', E)\,dE = \frac{gdE}{EE'}\left[1 + \left(\frac{E}{E'}\right)^2\right] = \frac{gdE}{E'E}\,[1 + \delta\,(E'E)],$$

in which it was assumed that $\delta(E'E)$ is small compared to unity. The correction $\kappa(E)$ to the approximate "equilibrium" spectrum of photons $\Gamma(E)$, given in reference 1 [Sec. 17, Eq. (17.8)], was calculated only for air in reference 2.

In the Table there are presented the results of the calculation of the equilibrium photon spectrum $\Theta(E) = \Gamma(E) + \kappa(E)$ for hydrogen, aluminum, iron, copper and lead. The calculation was carried out according to formulas obtained in reference 2. From these data it is evident that, as was to be expected, the corrections are large for heavy elements in the energy region of the order of the critical energy or lower.

The following values of critical energy were used in the present work: for hydrogen 120, for aluminum 37.2, for iron 18.4, for copper 22.4, for lead 6.4 mev. The results of the calculations are given in the Table.

From a comparison of the spectra for a given energy for hydrogen, aluminum, iron, copper and lead, it is easy to see that $\Theta(E)$ increases with increasing Z. Copper is an "exception" (compare with Fe). However, this result is connected with the choice of an inaccurate value of the critical energy given in the review of Rossi and Greisen[3], which was made on the basis of calculations for copper.

It should be noted that the correction can be regarded as small up to energies $\sim 0.1\beta$ or up to $Z = 30$, where the corrections for $\Gamma(E)$ amount to less than 10%. However, for Pb, the correction at 0.5 β is already 10.7 / 14.4 or 74.5%, and it is impossible to speak of the complete applicability of the method used in reference 2.

Comparison of the spectrum obtained in the present work for Pb with the spectrum obtained by Richards and Nordheim[4] indicates that, in contrast to the spectrum for air (for the case $E_m = \infty$, see reference 2), where the discrepancy of the spectra has been observed only in the region $E = 1$ because of the doubtful approximation carried out in reference 4 at this point, for lead the divergence is obtained for all energies $E < 2$. This comparison shows that the method set forth by Rossi and Greisen cannot give the correct result in the calculation of the photon spectrum of heavy elements at energies $E \sim 1.5\beta$ and lower.

In conclusion I express my gratitude to S. Z. Belen'kii for supervision of this research, and also to L. Ia. Zhil'tsov who carried out the principal calculations.

Translated by D. G. Posin
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The Gravitational Self Energy of Particles in the Classical Field Theories

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In several classical field theories[1,2] the electron is assigned an exclusively field self-energy. This energy can be calculated by a common formula[3], which allows the electron's own magnetism to be taken into account:

$$\mathcal{E}_0 = \int \varepsilon_0\,(\mathbf{E}, \mathbf{H})\,\sqrt{{}^1/_4\,(\mathbf{H}^2 - \mathbf{E}^2)^2 + (\mathbf{E}, \mathbf{H})^2}\,(dx), \qquad (1)$$

where **E**, **H** are the vector field intensities, ϵ_0 is the dielectric permeability of the vacuum. The result (without considering the magnetic moment) turns out to be $\mathcal{E}_0 = (1.2361\ \ldots)\,(e/x_0)$ in the Born-Infeld theory[1] and $\mathcal{E}_0 = e/2x_0$ in the theory of Bopp-Podolsky[2] where x_0 is the classical radius of the electron.

The question of the form of the linear gravitational field of the electron with a field mass is usually not considered, although it is a matter of considerable interest. We shall first calculate the gravitational potential of a point-electron with an infinite filled mass, making use of the relations

$$\square^2 h_{\mu\nu} = -(2G/c^3)\, T_{\mu\nu}, \quad (2)$$

where

$$T_{ik} = T_{i4} = 0, \qquad T_{44} = (e^2/2)\, x^{-4}. \quad (3)$$

The potential h_{44} takes the form

$$h_{44} = (Ge^2/c^3)\,[(1/2x^2) - (1/x_0 x)], \quad (4)$$

where x_0 is a constant small length, which it is necessary to introduce, in order to avoid the general divergence of h_{44}. At zero h_{44} now diverges as x^{-2}. The self-energy of the gravitational field is $\sim (\nabla h_{44})^2$ and diverges as x^{-6}. It is interesting, however, that by identifying h_{44} with the Newtonial potential, we obtain for x_0 the interpretation of classical radius $x_0 \doteq e^2/2\mathcal{E}_0$.

In the regulated theories we obtain the following results. In the theory of Born-Infeld*

$$\mathbf{D} = (e/x^2)\,\mathbf{x}^0, \quad \mathbf{E} = \mathbf{D}/\sqrt{1+(x/x_0)^4}, \quad (5)$$

and the gravitational potential is equal to

$$h_{44} = (Ge^2/c^3)\, F(\varphi, k)/2x_0 x$$
$$-(1/4x_0^2)\ln[(\sqrt{1+(x/x_0)^4}-1)/(\sqrt{1+(x/x_0)^4}+1)],$$

where $F(\phi, k)$ is the elliptic integral, $k = \sqrt{2}/2$,

$$\varphi = \arccos((1 - x^2/x_0^2)/(1 + x^2/x_0^2)).$$

For $x \gg x_0$, $h_{44} \sim 1/x$; for small distances $h_{44} \sim \ln x$ and diverges at zero. The gravitational self-energy converges and constitutes the portion of the self-energy of the electron required by the theory, as is easy to show,

$$m_{gr}/m_0 = aG^2 m_0^2/\hbar c,$$

where a is a number ~ 1.

In the theory of Bopp-Podolsky the result turns out to be divergent both for h_{44} and for m_{gr}. Thus only the non-linear electrodynamics allows a completely successful solution of the problem of the gravitational self energy. The result turns out to be favorable, notwithstanding the use of the linear theory of gravitation in calculation carried out above.

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* The question of the gravitational field of the electron in the theory of Born and Infeld was investigated by Fisher[4], who found exact solutions of the Einstein equations for this case.

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The Constriction and Curvature of an Arc in Rarified Gases at Large Currents

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THE idea of the constriction of the column of an electric arc in rarefied gases at large currents under the influence of its own magnetic field (the "pinch-effect") was introduced by Tonks[1]. It was shown that this hypothetical effect could cause a rupture of the arc.

In attempts to observe this phenomenon experimentally in a DC arc (for a Hg-arc at $i = 150$ amp), Thoneman and Cowhig[2] detected only a weakening of the radial electric field, while Mamyrin[3] (for a H_2 arc at $i \sim 100$ amp) found a constriction of the current density distribution by only 30%.

We performed experiments with DC arcs in rarefied Hg-vapor and inert gases in straight cylindrical tubes without constriction. Measurements with a movable probe in a tube of 20 mm diameter in Hg-vapor at $p = 1\,\mu$Hg confirmed that with an increase in the current from 1 to 80 amp the half-width* of the column is diminished by $\sim$25%. For a further increase in current to 1-2 kamp, it was impossible to find the corresponding constriction of the arc with probes, as firstly the probe currents, then the tension of the arc, and finally the current of the arc show strong, ever increasing irregular variations with a predominant frequency of the order of $10^4 - 10^5$ cps. Experiments with two identical probes, placed symmetrically with respect to the axis, showed that the current variations in both probes are completely uncorrelated in time: not only the position of the peaks in current but their number and relative height are different. Therefore, there occur in the tube no overall changes of the current intensity in the entire cross section of the column simultaneously, but probably fast irregular motions of the constricted channel of the arc take place over the cross section of the tube. As the arc moves