where A does not depend on  $\rho_1$ ,  $\rho_2$ ,  $\rho_3$  and b =  $G_{11} + G_{22} + G_{33}$ .

Direct evaluation of the coefficients  $\rho_2$ ,  $\rho_3$  and  $\rho_1$  in the expression  $F(r_1, r_2, r_3)$  shows that they are equal to zero and that

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$$F(r_1, r_2, r_3) = F(r_2, r_3, r_1) = F(r_3, r_1, r_2) = A.$$

Consequently, the determinant on the right side of Eq. (8) is equal to zero, and since  $\Delta \neq 0$ ,  $L_{12} = L_{21}$ . In a similar fashion it can be shown that  $L_{13} = L_{31}$ .

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# On the Theory of the Hall and Nernst-Ettinghausen Effects in Semiconductors with Mixed Conductivities

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The Hall and the Nernst-Ettinghausen voltages and the distribution of current carrier concentrations are calculated for a semiconductor with mixed conductivities located in a non-homogeneous magnetic field. Recombination of current carriers and energy levels due to impurities are taken into account.

>

### 1. INTRODUCTION

Some papers<sup>1-3</sup> have recently been published in which the effect of body and surface recombinations on the Hall effect in semiconductors has been investigated. The basic assumption in all these works has been Ohm's law in a form valid only for semiconductors without impurities, a form in which the mean free path of the current carriers does not depend on their velocity.

This work investigates both the Hall effect and the thermomagnetic Nernst-Ettinghausen effect under the assumption of a more general dependence of the length l of the mean free path of the current carriers on the velocity v:

$$l = \Phi(T) v^n, \tag{1}$$

where  $\Phi(T)$  is some function of the temperature T, and n is any given number. It will be shown that such a more general dependence of l on v leads to substantial quantitative changes in both the current carrier concentrations and the voltages in both effects, as well as to qualitative changes in the case of the Nernst-Ettinghausen effect. By analyzing the general case of a non-homogeneous magnetic field it is possible to determine how the non-uniformity of the magnetic field near the edges

<sup>1</sup> H. Welker, Z. Naturforsch, 6a, 184 (1951)

<sup>2</sup> A. I. Ansel'm, Zh. Tekhn. Fiz. 22, 1146(1952)

<sup>3</sup> R. Landauer and J. Swanson, Phys. Rev. 91, 207 (1953) of pole-pieces affect the phenomena under consideration.

The following assumptions are made:

1. The magnitude of the exponent n in Eq. (1) has the same value for both electrons and holes. This is equivalent to assuming an identical scattering process for both electrons and holes by the phonons.

2. The primary current (either electric or thermal) is directed along the x axis, and the magnetic field, which is a function of y, is directed along the z axis  $(H = H_z)$ . Therefore the electric fields of both the Hall and the Nernst-Ettinghausen effects are functions of y only.

### 2. BASIC EQUATIONS

The generalized differential equation of Ohm's law for electrons and holes, as derived by Tolpygo<sup>4</sup>, can be written:

$$\mathbf{j}_{+} = -eu_{+}N_{+}\left\{-\mathbf{E} + \frac{kT}{e}\right]$$
(2a)  
 
$$\times \left[\nabla \ln N_{+} + \frac{n+1}{2}\nabla \ln T\right] + \frac{a_{n}u_{+}}{c}.$$
  
 
$$\left\{-\mathbf{E} + \frac{kT}{e}\left(\nabla \ln N_{+} + \frac{n+1}{2}\nabla \ln T\right) \times \mathbf{H}\right\};$$
  
 
$$\mathbf{j}_{-} = -eu_{-}N_{-}\left\{-\mathbf{E}\right]$$
(2b)  
 
$$-\frac{kT}{e}\left[\nabla \ln N_{-} + \frac{n+1}{2}\nabla \ln T\right]$$

<sup>4</sup> K. B. Tolpygo, Trans. of Inst. of Phys, Acad. Sci. USSR 3, 52 (1952)

$$-\frac{a_n u_-}{c} \Big[ -\mathbf{E} - \frac{kT}{c} \Big( \nabla \ln N_- + n \nabla \ln T \Big) \Big] \times \mathbf{H} \Big\}.$$

The total current density is:

$$\mathbf{j} = \mathbf{j}_{+} + \mathbf{j}_{-} = -e\left\{-A_{1} \mathbf{E} + \frac{kT}{e} \left(\nabla A_{2} + A_{3} \nabla \ln T\right) + \left[-A_{4} \mathbf{E} + \frac{kT}{e} \left(\nabla A_{5} + A_{6} \nabla \ln T\right)\right] \times \frac{\mathbf{H}}{c}\right\}.$$
(2c)

where  $\underline{j}_{+}$ ,  $\underline{j}_{-}$  are current densities (the plus and minus subscripts designating holes and electrons respectively),  $N_{+}$ ,  $N_{-}$  the concentrations,  $m_{\pm}$  the effective masses, *e* the absolute value of charge on the electron, II the strength of the magnetic field,  $u_{\pm}$  the mobilities, which are

$$u_{\pm} = \frac{4}{3\sqrt{\pi}} \frac{e}{m_{\pm}} \Phi\left(T\right) \left(\frac{2kT}{m_{\pm}}\right)^{(n-1)/2} \times \Gamma\left(\frac{n}{2} + 2\right)$$

where

$$\Gamma(n) = \int_{0}^{\infty} x^{n-1} e^{-x} dx,$$

and

$$a_n = \frac{3\sqrt{\pi}}{4} \frac{\Gamma(n+(3|2))}{\Gamma^2((n/2)+2)};$$
(3)

$$A_{1} = u_{+} N_{+} + u_{-}N_{-}, \quad A_{2} = u_{+} N_{+} - u_{-}N_{-},$$
  

$$A_{3} = \frac{n+1}{2} A_{2}, \quad A_{4} = a_{n} (u_{-}^{2} N_{+} - u_{-}^{2} N_{-}),$$
  

$$A_{5} = a_{n} (u_{+}^{2} N_{+} + u_{-}^{2} N_{-}), \quad A_{6} = nA_{5}.$$

The equations of continuity for hole and electron currents in a semiconductor having energy levels due to impurities (see diagram) can be written:



Energy level diagram in a semiconductor.  $\mu$  is Fermi level.

$$\frac{1}{e} \operatorname{div} \mathbf{j}_{+} = \frac{1}{e} \operatorname{div} \mathbf{j}_{-} = k_{21} - k_{12}N_{+}N_{-} \qquad (4)$$

$$+ M b \frac{k_{31}k_{23} - k_{13}k_{32}N_{+}N_{-}}{k_{31} + k_{23} + k_{13}N_{-} + k_{32}N_{+}}$$

$$+ M_{a} \frac{k_{41}k_{24} - k_{14}k_{42}N_{+}N_{-}}{k_{41} + k_{24} + k_{14}N_{-} + k_{42}N_{+}},$$

where  $M_a$ ,  $M_d$  are densities of acceptors and donors respectively, and  $k_{ij}$  are kinetic coefficients with indices *i* and *j* denoting the type of transition. The kinetic coefficients have the following correlations<sup>5</sup>:

$$\frac{k_{21}}{k_{12}} = N_{+}^{0} N_{-}^{0}; \frac{k_{31}}{k_{13}} = \frac{N_{-}^{0} (M_{b} - N_{b}^{0})}{N_{b}^{0}}; \quad (5)$$
$$\frac{k_{41}}{k_{14}} = \frac{N_{-}^{0} N_{a}^{0}}{M_{a} - N_{a}^{0}}; \frac{k_{23}}{k_{32}} = \frac{N_{+}^{0} N_{b}^{0}}{M_{b} - N_{b}^{0}}.$$
$$\frac{k_{21}}{k_{42}} = \frac{N_{+}^{0} (M_{a} - N_{a}^{0})}{N_{a}^{0}}; \frac{k_{31}}{k_{43}} = \frac{(M_{b} - N_{b}^{0}) (M_{a} - N_{a}^{0})}{N_{b}^{0} N_{a}^{0}}$$

where  $N^0_+$  and  $N^0_-$  are the equilibrium densities of holes and electrons in the upper and lower zones, and  $N^0_a$ ,  $N^0_d$  the equillibrium densities of holes and

electrons at the acceptor and donor levels.

It will now be assumed that extraneous magnetic fields cause small divergences from equillibrium in the concentrations of holes and electrons, and that these divergences are equal for both holes and electrons:

$$N_{+} - N_{+}^{0} = N_{-} - N_{-}^{0} = \gamma; \quad \gamma / N_{\pm}^{0} \ll 1.$$
 (6)

Applying Eq. (5), Eq. (4) becomes

$$\operatorname{div} \mathbf{j}_{+} = \operatorname{div} \mathbf{j}_{-} = -e\alpha \nu \left(N_{+}^{0} + N_{-}^{0}\right), \qquad (4a)$$

where

$$\alpha = k_{12} + \frac{k_{13}k_{32}M_{b}}{k_{31} + k_{23} + k_{13}N_{-}^{0} + k_{32}N_{+}^{0}} + \frac{k_{14}k_{32}M_{a}}{k_{11} + k_{21} + k_{14}N_{-}^{0} + k_{42}N_{+}^{0}}.$$
(7)

The first component of Eq. (7), representing the recombination of holes and electrons in the upper and lower zones, predominates over the other components at high temperatures where intrinsic conductivity plays the main role. This value of  $a = k_{12}$  was used by Ansel'm in his investigations of semiconductors without impurities. At low

<sup>&</sup>lt;sup>5</sup> A. I. Gubanov, J. Exper. Theoret. Phys. USSR **21**, 79 (1951)

temperatures, where conductivity is determined by thermal transitions between the impurity energy levels and the upper and lower zones, the other two components are dominant in determining the value of *a*. This case, apparently, rarely occurs in practice.

## ISOTHERMAL HALL EFFECT

As is well known, the isothermal Hall effect (for  $\nabla T = 0$  in the specimen) consists of the fact that a so-called Hall voltage  $(V^x)$  appears in a semiconductor along the y axis when the semiconductor with current flowing along its x axis is placed in a transverse magnetic field directed along the z axis. For no current flowing along the y axis  $(\dot{J}_y = 0)$ , the value of the Hall electric field  $(E^x)$  is :

$$E^{\mathbf{X}} = \frac{kT}{e} \frac{u_{+} - u_{-}}{N_{+}^{0} u_{+} + N_{-}^{0} u_{-}} \frac{dv}{dy}$$

$$+ \frac{a_{n}}{ec} \frac{N_{+}^{0} u_{+}^{2} + N_{-}^{0} u_{-}^{2}}{(N_{+}^{0} u_{+} + N_{-}^{0} u_{-})^{2}} j_{\mathbf{X}} H.$$
(8a)

The Hall voltage is then:

$$V^{\mathbf{X}} = \int_{0}^{a} E^{\mathbf{X}} dy = \frac{kT}{e} \frac{u_{+} - u_{-}}{N_{+}^{0} u_{+} + N_{-}^{0} u_{-}} [\gamma(a) \quad (8b)$$
$$-\gamma(0)] + \frac{a_{n}}{ec} \frac{N_{+}^{0} u_{+}^{2} - N_{-}^{0} u_{-}^{2}}{(N_{+}^{0} u_{+} + N_{-}^{0} u_{-})^{2}} \times j_{\mathbf{X}} \int_{0}^{a} H dy,$$

where a is the length of the specimen along the y axis.

The Hall coefficient in the case of an arbitrary n is:

$$R = \frac{a_n}{ec} \frac{N_+ u_+^2 - N_- u_-^2}{(N_+ u_+ + N_- u_-)^2}.$$
 (9)

As can be seen from Table 1, the choice of  $a_n$  can cause the value of the Hall coefficient to change by as much as 1.5 to 2 times. Combining Eqs. (2), (5), (8a) and (4a) we can state the equation for charge distribution with accuracy to the order of  $\nu H/c$  by :

$$\frac{d^2\nu}{dy^2} - \frac{1}{\lambda_0^2} \,\nu = -f^X \frac{dH}{dy} \,. \tag{10}$$

where

$$f^{\mathbf{X}} = \frac{a_n}{c} \frac{e}{kT} \frac{N_+^0 N_-^0 (u_+ + u_-)}{N_+^0 + N_-^0} E_{0}$$

 $\begin{array}{c} E_{0} \text{ is the strength of the electric field along the } x \\ \text{axis, and } \lambda_{0}^{=} \sqrt{\frac{kT}{c} \frac{u_{+}u_{-}}{\alpha \left(N_{+}^{0}u_{+} + N_{-}^{0}u_{-}\right)}} \end{array} \end{array}$ 

is the current carrier's average diffusion length.

TABLE I

Values of coefficients  $a_n$ for various  $n^4$ 

n	• <sup>a</sup> n	n	<sup>a</sup> n
-0.5 0.5 1.0 1.5	$\begin{array}{c} 1.5740 \\ 1.1781 \\ 1.0356 \\ 1.0000 \\ 1.0277 \end{array}$	$2.0 \\ 2.5 \\ 3.0 \\ 3.5 \\ 4.0$	1,1043 1,2276 1,4003 1,6308 1,9328

When surface energy levels and surface recombination are present in a semiconductor it is possible to obtain the boundary conditions of the problem by applying Eq. (10) to a case of an infinitely thin layer of thickness  $\delta$ :

$$\frac{dv}{dy} - \frac{v}{\lambda_s} = -f^X H \qquad \text{for } y = 0 \qquad (11)$$

$$\frac{dv}{dy} + \frac{v}{\lambda_s} = - \frac{x}{\lambda}H \qquad \text{for } y = a$$

Here a new characteristic length is introduced:  $\lambda_s = \lim_{\delta \to 0} (\lambda_0^2 / \delta)$ 

Isothermal Nernst-Ettinghausen Effect

The isothermal Nernst-Ettinghausen effect  $(\partial T/\partial y = 0)$  consists of the appearance of a potential difference in a semiconductor along the y axis when its x axis, along which exists a temperature gradient, is placed in a transverse magnetic field along the z axis.

For  $j_x = j_y = 0$ , the Nernst-Ettinghausen field  $E_{\rm H}$  is:

$$E^{\rm H} = \frac{kT}{e} \left\{ \frac{u_{+} - u_{-}}{N_{+}^{0} u_{+} + N_{-}^{0} u_{-}} \frac{dv}{dy}$$
(12a)  
+  $\frac{a_{n}}{c} \left[ \frac{1 - n}{2} \frac{N_{+}^{02} u_{+}^{3} + N_{-}^{02} u_{-}^{3}}{(N_{+}^{0} u_{+} + N_{-}^{0} u_{-})^{2}} - \frac{3n + 7}{2} \frac{N_{+}^{0} N_{-}^{0} u_{+} u_{-} (u_{+} + u_{-})}{(N_{-}^{0} u_{+} + N_{-}^{0} u_{-})^{2}} \times \left( 1 + \frac{2}{3n + 7} \frac{\Delta E}{kT} \right) H \frac{d \ln T}{dx} \right\}.$ 

and the Nernst-Ettinghausen voltage V<sup>H</sup> is:

$$V^{\rm H} = \int_{0}^{a} E^{\rm H} dy = \frac{kT}{c} \left\{ \frac{u_{+} - u_{-}}{N_{+}^{0} u_{+} + N_{-}^{0} u_{-}} \right.$$
(12b)  
 
$$\times [\gamma(a) - \gamma(0)] + \frac{a_{n}}{c} \left[ \frac{1 - n}{2} \times \frac{N_{+}^{0^{2}} u_{+}^{3} + N_{-}^{0^{2}} u_{-}^{3}}{\left(N_{+}^{0} u_{+} + N_{-}^{0} u_{-}\right)^{2}} \right]$$

n

$$-\frac{3n+7}{2}\frac{N_{+}^{0}N_{-}^{0}u_{+}u_{-}(u_{+}+u_{-})}{(N_{+}^{0}u_{+}+N_{-}^{0}u_{-})^{2}}\left(1+\frac{2}{3n+7}\frac{\Delta E}{kT}\right)\right]$$
$$\times\frac{d\ln T}{dx}\int_{0}^{a}H\,dy\Big\}\cdot$$

In expressions (12a) and (12b)  $\Delta E$  denotes the width of the forbidden zone. The Nernst-Etting-hausen coefficient, in the case of an arbitrary *n*, is:

$$Q = \frac{k}{e} \frac{a_n}{c} \left[ \frac{1-n}{2} \frac{N_+^2 u_+^3 + N_-^2 u_-^3}{(N_+ u_+ + N_- u_-)^2} \right]$$
(13)

$$-\frac{3n+7}{2} \frac{N_{+}N_{-}u_{+}u_{-}(u_{+}+u_{-})}{(N_{+}u_{+}+N_{-}u_{-})^{2}} \times \left(1+\frac{2}{3n+7}\frac{\Delta E}{kT}\right)].$$

The charge distribution is given by

$$\frac{d^2 \nu}{dy^2} - \frac{1}{\lambda_0^2} \nu = -f^{\mathsf{H}} \frac{dH}{dy}, \qquad (14)$$

where

$$f^{\rm H} = \frac{a_n}{c} \frac{d\ln 7}{dx} \tag{15}$$

$$\left( N^{0}_{+} N^{0}_{-} \left[ \frac{1-n}{2} \left( N^{0}_{+} u^{2}_{+} - N^{0}_{-} u^{2}_{-} \right) + \frac{3n+7}{2} \right] \right)$$

$$\times u_{+}u_{-} \left( N^{0}_{+} - N^{0}_{-} \right) \left( 1 + \frac{2}{3n+7} \frac{\Delta E}{kT} \right) \right)$$

$$\times \frac{1}{\left( N^{0}_{+} + N^{0}_{-} \right) \left( N^{0}_{+} u_{+} + N^{0}_{-} u_{-} \right)} ,$$

$$\frac{dv}{dy} - \frac{v}{\lambda_{s}} = -f^{H}H \quad \text{for } y = 0$$

$$\frac{dv}{dy} + \frac{v}{\lambda_{s}} = -f^{H}H \quad \text{for } y = a$$

Thus the investigation of both effects is reduced to solving equations of the type (10) and (14) with boundary conditions of the type (11) and (15).

### 3. SOLUTION OF EQUATIONS AND ANALYSIS OF RESULTS

The solution of Eqs. (10) and (14) with boundary conditions (11) and (15) has the form:

$$v^{X} = f^{X}F(y), v^{H} = f^{H}F(y),$$
 (16)

where

$$F(y) = \frac{\lambda_0}{\left(1 + \frac{\lambda_0^2}{\lambda_s^2}\right) \operatorname{sh} \frac{a}{\lambda_0} + 2\frac{\lambda_0}{\lambda_s} \operatorname{ch} \frac{a}{\lambda_0}} \qquad (17)$$
$$\times \left[H(0)\left(\operatorname{ch} \frac{y-a}{\lambda_0} - \frac{\lambda_0}{\lambda_s} \operatorname{sh} \frac{y-a}{\lambda_0}\right)\right]$$

$$+ \left( \int_{0}^{\infty} \operatorname{ch} \frac{a - y_{1}}{\lambda_{0}} \frac{dH}{dy_{1}} dy_{1} \right. \\ + \frac{\lambda_{0}}{\lambda_{s}} \int_{0}^{a} \operatorname{sh} \frac{a - y_{1}}{\lambda_{0}} \frac{dH}{dy_{1}} dy_{1} - H(a) \right) \\ \times \left( \operatorname{ch} \frac{y}{\lambda_{0}} + \frac{\lambda_{0}}{\lambda_{s}} \operatorname{sh} \frac{y}{\lambda_{0}} \right) \left[ - \lambda_{0} \int_{0}^{y} \operatorname{sh} \frac{y - y_{1}}{\lambda_{0}} \frac{dH}{dy_{1}} dy_{1} \right]$$

It is now possible to determine in the equations for the Hall and the Nernst-Ettinghausen voltages the components that are dependent on the charge distribution.

These components,  $V_1^X$  (for Hall effect) and  $V_1^H$  (for Nernst-Ettinghausen effect) are:

$$V_{1}^{X} = \frac{kT}{e} \frac{u_{+} - u_{-}}{N_{+}^{0} u_{+} + N_{-}^{0} u_{-}} f^{X} [F(a) - F(0)];$$
 (18a)

$$V_{1}^{\mathsf{H}} = \frac{kT}{e} \frac{u_{+} - u_{-}}{N_{+}^{0} u_{+} + N_{-}^{0} u_{-}} f^{\mathsf{H}} [F(a) - F(0)].$$
(18b)

where, from Eq. (17)

$$F(a) - F(0) = \frac{\lambda_0}{\left(1 + \frac{\lambda_0^2}{\lambda_s^2}\right) \operatorname{sh} \frac{a}{\lambda_0} + 2\frac{\lambda_0}{\lambda_s} \operatorname{ch} \frac{a}{\lambda_0}} \quad (19)$$

$$\times \left[H(0)\left(1 - \operatorname{ch} \frac{a}{\lambda_0} - \frac{\lambda_0}{\lambda_s} \operatorname{sh} \frac{a}{\lambda_0}\right) + \left(\int_0^a \operatorname{ch} \frac{a - y_1}{\lambda_0} \frac{dH}{dy_1} dy_1 + \frac{\lambda_0}{\lambda_0} \int_0^a \operatorname{sh} \frac{a - y_1}{\lambda_0} \frac{dH}{dy_1} dy_1 - H(a)\right)$$

$$\times \left(\operatorname{ch} \frac{a}{\lambda_0} + \frac{\lambda_0}{\lambda_s} \operatorname{sh} \frac{a}{\lambda_0} - 1\right) - \lambda_0 \int_0^a \operatorname{sh} \frac{a - y_1}{\lambda_0} \frac{dH}{dy_1} dy_1.$$

It is apparent from Eq. (16) that  $\nu^{\chi}$  is always positive; that is, that the current carrier concentration can only increase in the case of the Hall effect. On the other hand,  $\nu^{\text{H}}$  in the case of the Nernst-Ettinghausen effect, can have either sign. In the case of semiconductors having holes and electrons of equal mobilities the potential differences  $V_1$  and  $V_1$  are equal to zero at the same time that  $\nu^{\chi}$  and  $\nu^{\text{H}}$  differ from zero.

In the Nernst-Ettinghausen effect, the change in concentration becomes zero when either of the following two conditions is fulfilled:

1)  $N_{+}^{0} = N_{-}^{0}$ ,  $u_{+} = u_{-}$ , for n = any number; 2)  $N_{+}^{0} = N_{-}^{0}$ , for n = 1

It is interesting to consider the limiting case when the diffusion length  $\lambda_0 \rightarrow \infty (\alpha \rightarrow 0)$ , and  $N_{+} = N_{-} = N$ . Eq. (4a) can then be solved uniquely.  $\vdots$  for the Nernst-Ettinghausen effect The equations and boundary conditions now become:

$$\frac{d^2N}{dy^2} + b \frac{dHN}{dy} = 0;$$
(20)

$$\frac{dN}{dy} + bHN = \frac{N - N^0}{\lambda_s} \quad \text{for } y = 0$$
 (21)

$$\frac{dN}{dy} + bHN = -\frac{N-N^0}{\lambda_s}$$
 for  $y=a$ 

For the Hall effect

$$b = b^{\mathbf{X}} = \frac{a_n e \left(u_+ + u_-\right)}{2ckT} E_0.$$

For the Nernst-Ettinghausen effect

$$b = b^{\mathrm{H}} = \frac{1 - n}{2} \frac{u_{+} - u_{-}}{c} \frac{d \ln 7}{dx}$$

Solving (20) with boundary conditions (21), we get

$$N(y) = \frac{N^{0}}{1 + w^{-1}(a) + \lambda_{s}^{-1}w^{-1}(a)\int_{0}^{a}w(y)dy}$$
(22)  

$$\times \left( \left[ 2 + \lambda_{s}^{-1}w^{-1}(a)\int_{0}^{a}w(y)dy \right]w^{-1}(a) + \left[ 1 - w^{-1}(a) \right]\lambda_{s}^{-1}w^{-1}(y)\int_{0}^{y}w(y)dy \right)$$
where  $w(y) = \exp(b\int_{0}^{y}Hdy).$ 

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The voltages  $V_1$  ( $V_1^X$  or  $V_1^H$ ) dependent on the change in the concentration of current carriers are:

$$V_{1} = \frac{kT}{e} \frac{u_{\perp} - u_{\perp}}{u_{+} + u_{-}}$$
(23)  
×  $\ln \frac{\left[2 + \lambda_{s}^{-1} \int_{0}^{a} w(y) \, dy\right] w^{-1}(a)}{2 + \lambda_{s}^{-1} w^{-1}(a) \int_{0}^{a} w(y) \, dy}.$ 

The extent to which the proposed theory is applicable will now be determined. To simplify matters, we will assume  $N_{\pm}^{0} = N_{\pm}^{0} = N$ ; and will let u stand for the greater of the magnitudes  $u_{\pm}$  and  $u_{\pm}$ . Using as a criterion of validity the magnitude of the discarded components in (10) and (14) we can say: for the Hall effect

$$\frac{\partial_{c_0} eF_0}{2kT} \frac{uH}{c} \ll 1;$$
 24a)

$$\frac{\lambda_0}{4} \frac{d\ln T}{dx} \frac{uH}{c} \ll 1.$$
 (24b)

Let us consider the case of a high conducting semiconductor of the germanium type. Let us assume u = 106 cgs units,  $\lambda_0 = 0.1$  cm, T = 300 %, and say the Hall effect is to be investigated at  $E_0 = 1 \ V/cm$  and  $H = 10^3$  oersteds. Then

$$\frac{\lambda_0 e E_0}{2kT} \frac{uH}{c} \approx 0.07 \ll 1.$$

For the Nernst-Ettinghausen effect, let us assume H to be  $10^4$  oersteds, and  $d \ln T/dx = 1 \text{ cm}^{-1}$  $\frac{\lambda_0}{4} \frac{uH}{c} \frac{d\ln T}{dx} \approx 0.008 \ll 1.$ Then

These estimates show that in the case of high conducting semiconductors with current carriers of high mobility, the Hall effect theory is valid for a temperature range not lower than room temperatures if at the same time the magnetic fields do not exceed 10<sup>3</sup> oersteds. The Nernst-Ettinghausen effect theory, on the other hand, holds good for a greater range of temperatures ( in the direction of lower temperatures ) and magnetic fields ( in the direction of stronger fields). In the case of semiconductors with current carriers of low mobility ( of Cu, O type) the range of applicability of both theories considerably increases.

We will now examine how the diffusion of the current carriers affects both their concentration and the strength of the Hall and the Nernst-Ettinghausen fields. For simplicity, we will assume that the magnetic field is uniform and that surface effects are absent  $(\lambda_s \rightarrow \infty)$ . Then

$$F(a) - F(0) \approx -2\lambda_0 H$$

and the relative change of concentration in the Hall effect is . . .

$$\left|\frac{v^{X}}{N^{0}}\right| \approx \frac{\lambda_{0}eE_{0}}{kT}\frac{uH}{c}.$$

For semiconductors of the germanium type having the above parameters, the relative change in concentration in the Hall effect will be about 13%, while the relative change in the Hall voltage due to diffusion (if we assume the length a along the  $\gamma$  axis of the specimen to be 0.5 cm) must reach 20% In the Nernst-Ettinghausen effect, the relative change in concentration as determined by the expression

$$\left|\frac{\mathbf{v}^{\mathrm{H}}}{N^{\mathrm{o}}}\right| \approx \frac{\lambda_{\mathrm{o}}d\ln T}{2dx}\frac{uH}{c}$$

should be, under the same conditions, about 2%, while the relative change in voltage should reach about 20%

Numerical calculations show that the non-uniformity of the magnetic field at the edges of pole – pieces can not cause any significant changes in the magnitude of the effects under consideration.

#### CONCLUSION

Analysis of Eqs. (8b), (12b), (18a) and (18b) shows that for both phenomena, when surface effects are missing,  $|(V - V_0)/V_0| \approx \lambda_0/a$ , , where V is the actual voltage determined experimentally, and  $V_0$  is the intrinsic Hall or Nernst-Ettinghausen voltage before inclusion of voltage effects due to changes in current carrier concentrations. From this it follows that the magnitudes of the Hall and the Nernst-Ettinghausen voltages will be determined with the same degree of exactness with which the inequality  $\lambda_0/a \ll 1$  is attained. When the condition that  $\lambda_0/a \ll 1$  is fulfilled experimentally, V will not differ from  $V_0$  If the length of the specimen  $a_1$  is far greater than its width  $a_2$ , and the latter is equal to several current carrier diffusion lengths  $\lambda_0$ , then  $\lambda_0$  can be evaluated by:

$$\lambda_0 \approx \frac{V(a_2)}{V(a_1)} a_1 - a_2,$$
 (26)

where  $V(a_1)$  and  $V(a_2)$  are the Hall or the Nernst-Ettinghausen voltages measured along the length and along the width respectively. Thus to evaluate  $\lambda_0$  it is sufficient to measure the voltage V across the specimen at two mutually perpendicular orientations of it. It follows from the statement at the conclusion of the preceeding section that equation (26) will hold true as well in the case of a specimen which is located in a field that is nonuniform at the edges.

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