Other phenomena*, observed in our previous experiments with tungsten at large current densities and not observed under ordinary conditions, depend considerably on the current density. These phenomena were considered by us as indications of an anomalous state of the tungsten caused by a flow of current of high density. The dependence of energy at the instant of explosion - $E(t_c)$ - on j and emission anomalies is observed in the experiments treated in present paper, not only on tungsten, but also on nickel. New peculiarities of emission (see point 4 of the Conclusions) were discovered which we also could not explain by means of a supposition that the metal is in a normal state . We note that the conditions of the experiments on nickel and on tungsten were materially different because of the considerable difference in melting point temperature (for example the measured anode currents were different by several orders of magnitude). The identical character of the phenomena observed in these metals near $R = R^s_{mp}$ shows that they are caused by processes in metals independent of the value of their melting point temperature.

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* The dependence on the energy $E(t_c)$ on j (t) [e.g., see S. V. Lebedev and S. E. Khaikin, J. Exper. Theoret. Phys. USSR **26**, 629 (1954) and S. V. Lebedev, J. Exper. Theoret. Phys. USSR **27**, 605 (1954)], anomalies of emission [e.g., see S. V. Lebedev and S. E. Khaikin, J. Exper. Theoret. Phys. USSR **27**, 487 (1954)], peculiarities of melting in experiments with interrupted pulses [e.g., see S. V. Lebedev, J. Exper. Theoret. Phys. USSR **27**, 605 (1954)].

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The Wave Function of the Lowest State of a System of Interacting Bose Particles

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The wave function and the energy of the lowest state of a slightly non-ideal Bose gas are determined by means of the method of " auxiliary variables ", with accuracy to terms of second order relative to the smallness of the parameter of the energy of interaction.

I. INTRODUCTION

THE problem of the investigation of the wave of a system of a large number of interacting Bose particles arose in connection with attempts at the formation of a microscopic theory of the superfluidity of He II. In spite of a series of successes in this direction[1,2] we are today still far from the completion of such a microscopic theory.

If we select as a model for the liquid helium a slightly non-ideal degenerate Bose gas, then it is possible , as one of us has shown[2], to explain the phenomenon of the superfluidity of He II by the properties of the energy spectrum of such a system. However, inasmuch as the slightly non-ideal Bose gas cannot be regarded an entirely satisfactory model of liquid helium, the necessity arises of improving the theory of the non-ideal Bose gas, taking into account interactions that are not small. Up to the present time only such systems with weak interactions between particles have been studied theoretically.

Wave functions of the lowest state of a system consisting of a large number of weakly interacting Bose particles have been determined by Bijl[3]. However the results of his work are in error because of the lack of validity of the approximations used (i.e., terms that are not small in magnitude have been neglected).

In the present work the correct wave functions of the lowest state of a Bose system with weak interaction have been determined by means of the method of "auxiliary variables" with accuracy to terms of second order of smallness.

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[1] L.D. Landau, J. Exper. Theor. Phys. USSR 11, 592 (1941).

[2] N.N. Bogoliubov, Izv. Akad. Nauk SSSR, Ser. Fiz. 11, 77 (1947).

[3] A. Bijl, Physica 7, 869 (1940).

II. APPLICATION OF THE METHOD OF AUXILIARY VARIABLES TO BOSE SYSTEMS

We consider N interacting particles, without spin that obey to Bose-Einstein statistics. To describe the system we introduce the variables $\rho_{\mathbf{k}}$, the Fourier coefficients of the density operator, according to the formula

$$\rho_{\mathbf{k}} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-i\mathbf{k}\cdot\mathbf{r}_j} \quad (\mathbf{k} \neq 0). \tag{2.1}$$

The quantity $\rho_0 = \sqrt{N}$ is a constant and cannot be employed as a variable. $1/\sqrt{N}$ is a normalizing factor.

The variables $\rho_{\mathbf{k}}$ appear to be natural "collective" variables for describing oscillatory processes in systems that consist of a large number of interacting particles, and were applied earlier to systems of Fermi particles in the works of Zubarev[4], Tomonaga[5], and Pines and Bohm[6]

We shall seek a wave function of the systems in the form

$$\varphi(t, \rho_{\mathbf{k}_1} \ldots \rho_{\mathbf{k}_i} \ldots), \tag{2.2}$$

which does not contain explicitly coordinates of the particles but only the auxiliary variables $\rho_{\mathbf{k}}$. This representation of the wave function we shall call the $\rho_{\mathbf{k}}$-representation in what follows. This is a feasible arrangement since the wave function of the system is symmetric relative to the coördinates $\mathbf{r}_1, \ldots, \mathbf{r}_N$ and $\rho_{\mathbf{k}}$ is also a symmetric function of the coördinates particles.

The number of variables in the wave function (2.2) is infinite, and they are not independent. In order to go over to the usual representation of the wave functions, it is necessary to substitute Eq. (2.1) for $\rho_{\mathbf{k}}$ in Eq. (2.2); the resultant function will be symmetric relative to $\mathbf{r}_j$.

The Schrödinger equation for a system of N particles has the form

$$i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m} \sum_{j=1}^{N} \Delta_{\mathbf{r}_j} \varphi \tag{2.3}$$

$$+ \frac{1}{2} \sum_{\substack{j_1, j_2 \\ j_1 \neq j_2}} \Phi(|\mathbf{r}_{j_1} - \mathbf{r}_{j_2}|) \varphi.$$

It is now required to put this equation in the $\rho_{\mathbf{k}}$-representation for a system of Bose particles.

We express the interaction operator in terms of the variables $\rho_{\mathbf{k}}$.:

$$\frac{1}{2} \sum_{j_1 \neq j_2} \Phi(|\mathbf{r}_{j_1} - \mathbf{r}_{j_2}|) \tag{2.4}$$

$$= \sum_{\mathbf{k} \neq 0} \frac{N}{2V} \nu(k) \rho_{\mathbf{k}} \rho_{-\mathbf{k}} + \frac{N^2}{2V} \nu(0) - \sum_{\mathbf{k}} \frac{N}{2V} \nu(k),$$

where $\nu(k) = \int \Phi(r) e^{-i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$ is the Fourier coefficient of the energy of interaction. We introduce the Bose-Einstein kinetic energy operator in the $\rho_{\mathbf{k}}$ representation, carrying out the differentiation of the function (2.2) as an implicit function of the $\mathbf{r}_j$:

$$T = \frac{1}{\sqrt{N}} \sum_{\substack{\mathbf{k}_1, \mathbf{k}_2 \\ \mathbf{k}_1 + \mathbf{k}_2 \neq 0}} \frac{\hbar^2}{2m} (\mathbf{k}_1 \mathbf{k}_2) \rho_{\mathbf{k}_1 + \mathbf{k}_2} \frac{\partial^2}{\partial \rho_{\mathbf{k}_1} \partial \rho_{\mathbf{k}_2}} \tag{2.5}$$

$$+ \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m} \left(-\frac{\partial^2}{\partial \rho_{\mathbf{k}} \partial \rho_{-\mathbf{k}}} + \rho_{\mathbf{k}} \frac{\partial}{\partial \rho_{\mathbf{k}}} \right).$$

Making use of the Eqs. (2.4), (2.5) for the potential and kinetic energy, we write the Schrödinger equation in the $\rho_{\mathbf{k}}$ representation in the form

$$i\hbar \frac{\partial \varphi}{\partial t} = \sum_{\mathbf{k} \neq 0} \left\{ \frac{\hbar^2 k^2}{2m} \left(-\frac{\partial^2 \varphi}{\partial \rho_{\mathbf{k}} \partial \rho_{-\mathbf{k}}} + \rho_{\mathbf{k}} \frac{\partial \varphi}{\partial \rho_{\mathbf{k}}} \right) \right. \tag{2.6}$$

$$\left. + \frac{1}{2} \frac{N}{V} \nu(k) \rho_{\mathbf{k}} \rho_{-\mathbf{k}} \varphi \right\} + \frac{1}{\sqrt{N}}$$

$$\times \sum_{\substack{\mathbf{k}_1, \mathbf{k}_2 \\ \mathbf{k}_1 + \mathbf{k}_2 \neq 0}} \frac{\hbar^2}{2m} (\mathbf{k}_1 \mathbf{k}_2) \rho_{\mathbf{k}_1 + \mathbf{k}_2} \frac{\partial^2 \varphi}{\partial \rho_{\mathbf{k}_1} \partial \rho_{\mathbf{k}_2}}$$

$$+ \left\{ \frac{N^2}{2V} \nu(0) - \frac{N}{2V} \sum_{\mathbf{k}} \nu(k) \right\} \varphi.$$

We have thus shown that if the function (2.2) is a solution of Eq. (2.6) in which the $\rho_{\mathbf{k}}$ are the independent variables, then the function

$$\varphi\left(t, \ldots \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \exp\{-i(\mathbf{k}\mathbf{r}_j)\}, \ldots\right) \quad \text{is a solu-}$$

tion of the Schrödinger equation, symmetric relative to $\mathbf{r}_1, \ldots, \mathbf{r}_N$. We can therefore work with Eq. (2.6) in what follows, regarding the $\rho_{\mathbf{k}j}$ as the independent variables.

The Hamiltonian operator in Eq. (2.6) is non-

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[4] D.N. Zubarev, J. Exper. Theor. Phys. USSR **25**, 548 (1953).

[5] S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950).

[6] D. Pines and D. Bohm, Phys. Rev. **82**, 625 (1951). **85**,338 (1952); **92**, 609(1953).

Hermitian, since the transformation to the variables ρ_k is not a canonical one. The principal part of the Hamiltonian can be made Hermitian if we introduce a new wave function Φ by the substitution

$$\Phi = \exp\Big\{-\frac{1}{4}\sum_{\mathbf{k}}\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\Big\}\varphi. \tag{2.7}$$

Then Eq. (2.6) takes the form

$$i\hbar\frac{\partial\Phi}{\partial t} = \Big\{\sum_{\mathbf{k}\neq 0}\Big[-\frac{\hbar^2k^2}{2m}\frac{\partial^2}{\partial\rho_{\mathbf{k}}\partial\rho_{-\mathbf{k}}} \tag{2.8}$$

$$+\frac{1}{2}\Big(\frac{N}{V}\nu(k)+\frac{\hbar^2k^2}{4m}\Big)\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\Big] + \frac{1}{\sqrt{N}}$$

$$\times\sum_{\substack{\mathbf{k}_1,\mathbf{k}_2\\ \mathbf{k}_1+\mathbf{k}_2\neq 0}}\frac{\hbar^2}{2m}(\mathbf{k}_1\mathbf{k}_2)\rho_{\mathbf{k}_1+\mathbf{k}_2}\Big(\frac{\partial}{\partial\rho_{\mathbf{k}_1}}+\frac{\rho_{-\mathbf{k}_1}}{2}\Big)\Big(\frac{\partial}{\partial\rho_{\mathbf{k}_2}}+\frac{\rho_{-\mathbf{k}_2}}{2}\Big)$$

$$-\sum_{\mathbf{k}}\Big(\frac{\hbar^2k^2}{4m}+\frac{N\nu(k)}{2V}\Big)+\frac{N^2}{2V}\nu(0)\Big\}\Phi.$$

Transforming to the variables $q_{\mathbf{k}}$,

$$\rho_{\mathbf{k}} = \sqrt{2}\lambda_{\mathbf{k}}q_{\mathbf{k}}, \tag{2.9}$$

$$\lambda_{\mathbf{k}}^4\Big(\frac{N}{V}\nu(k)+\frac{\hbar^2k^2}{4m}\Big) = \frac{\hbar^2k^2}{4m}, \tag{2.10}$$

and we obtain Eq. (2.8) in the form

$$i\hbar\frac{\partial\Phi}{\partial t} = \Big\{\frac{1}{2}\sum_{\mathbf{k}}E(k)\Big(-\frac{\partial^2}{\partial q_{\mathbf{k}}\partial q_{-\mathbf{k}}}+q_{\mathbf{k}}q_{-\mathbf{k}}\Big) \tag{2.11}$$

$$+\frac{i}{\sqrt{N}}\sum_{\substack{\mathbf{k}_1,\mathbf{k}_2\\ \mathbf{k}_1+\mathbf{k}_2\neq 0}}\frac{\hbar^2}{2m}(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\sqrt{2}\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}q_{\mathbf{k}_1+\mathbf{k}_2}\Big(\frac{\partial}{\partial q_{\mathbf{k}_1}}$$

$$+\lambda_{\mathbf{k}_1}^2q_{-\mathbf{k}_1}\Big)\Big(\frac{\partial}{\partial q_{\mathbf{k}_2}}+\lambda_{\mathbf{k}_2}q_{-\mathbf{k}_2}\Big)+\frac{N^2}{2V}\nu(0)$$

$$-\sum_{\mathbf{k}}\Big(\frac{\hbar^2k^2}{4m}+\frac{N}{2V}\nu(k)\Big\}\Phi,$$

where

$$E(k) = \sqrt{\frac{N}{V}\nu(k)\frac{\hbar^2k^2}{m}+\Big(\frac{\hbar^2k^2}{2m}\Big)^2} \tag{2.12}$$

We introduce the Bose operators

$$b_{\mathbf{k}} = \frac{1}{\sqrt{2}}\Big(\frac{\partial}{\partial q_{-\mathbf{k}}}+q_{\mathbf{k}}\Big), \tag{2.13}$$

$$b_{\mathbf{k}}^+ = \frac{1}{\sqrt{2}}\Big(-\frac{\partial}{\partial q_{\mathbf{k}}}+q_{-\mathbf{k}}\Big),$$

with the help of which the Schrödinger equation for a system of interacting Bose particles can be written in the form

$$i\hbar\frac{\partial\Phi}{\partial t} = \Big\{\sum_{\mathbf{k}}E(k)b_{\mathbf{k}}^+b_{\mathbf{k}}+\frac{1}{\sqrt{N}} \tag{2.14}$$

$$\times\sum_{\substack{\mathbf{k}_1,\mathbf{k}_2\\ \mathbf{k}_1+\mathbf{k}_2\neq 0}}\frac{\hbar^2}{8m}(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}(b_{\mathbf{k}_1+\mathbf{k}_2}+b^+_{-\mathbf{k}_1-\mathbf{k}_2})$$

$$\times[(1+\lambda_{\mathbf{k}_1}^2)b_{-\mathbf{k}_1}+(\lambda_{\mathbf{k}_1}^2-1)b_{\mathbf{k}_1}^+][(1+\lambda_{\mathbf{k}_2}^2)b_{-\mathbf{k}_2}$$

$$+(\lambda_{\mathbf{k}_2}^2-1)b_{\mathbf{k}_2}^+]\Big\}\Phi,$$

where the constant E_0,

$$E_0 = \frac{N^2}{2V}\nu(0)+\frac{1}{2}\sum_{\mathbf{k}}\Big(E(k)-\frac{\hbar^2k^2}{2m}-\frac{N}{V}\nu(k)\Big) \tag{2.15}$$

has been eliminated by means of the substitution

$$\Phi\to\Phi\exp\{E_0t/i\hbar\}.$$

Thus the Hamiltonian of a system of interacting Bose particles consists of two parts: the diagonal part, which represents the sum of the Bose density fluctuation operators (or elementary phonon excitations) with energy $E(k)$, and a non-diagonal part, that describes the phonon interaction. The energy of the elementary excitations in a Bose gas agrees with the obtained earlier[2].

The first sum in Eqs. (2.6), (2.8), (2.11), (2.14) represents the principal part of the Hamiltonian, the second sum is the secondary (and, in general, small) part of the Hamiltonian. If the interaction between phonons tends to zero, then $\lambda_{\mathbf{k}}^2\to 1$. In this event, sizeable terms remain in the second sum of Eq. (2.14). These terms contain the operators $(b_{\mathbf{k}_1+\mathbf{k}_2}+b^+_{-\mathbf{k}_1-\mathbf{k}_2})b_{-\mathbf{k}_1}b_{-\mathbf{k}_2}$. However, inasmuch as we are interested only in the wave function of the lowest state, this term can be omitted, since it has no effect on the wave function of the vacuum.

3. THE RELATION BETWEEN THE METHOD OF "AUXILIARY VARIABLES" AND THE METHOD OF BIJL

Bijl's method[3] consists of the following. An approximate wave function of the lowest level of a system of N Bose particles is required which satisfies the Schrödinger equation (2.3). The interaction energy between the particles is assumed to be small, proportional to a parameter ϵ We choose as the zeroth approximation a uniform particle distribution density. The usual method in the theory of excitation, in which a decomposition of the desired wave function is expressed in a series of powers of ϵ, is not suitable here, since the corrections to the wave function of the zeroth approximation turn out not to be small, and to be proportional to a positive power of the total number of particles, N. For sufficiently large N, the correction can therefore be arbitrarily large. Bijl showed that it is possible to avoid this difficulty by expressing the logarithm of the wave function in powers of ϵ, rather than the wave function itself.

Setting

$$\varphi = e^{S}, \tag{3.1}$$

Bijl sought a new unknown function S in a power series in ϵ. We shall attempt to calculate S without making use of this decomposition. This precaution is necessary, because the coefficients for the neglected terms, which are proportional to ϵ^2, are not small, and cannot be disregarded.

If we substitute (3.1) in Schrödinger's equation we obtain the following for the logarithm of the wave function:

$$-\frac{\hbar^2}{2m}\sum_{j=1}^{N}\Delta_j S-\frac{\hbar^2}{2m}\sum_{j=1}^{N}(\nabla_j S)^2 \tag{3.2}$$

$$+\frac{1}{2}\sum_{j_1\neq j_2}\Phi(|\mathbf{r}_{j_1}-\mathbf{r}_{j_2}|)=E.$$

In this equation, Bijl neglected the second term (proportional to ϵ^2); we keep this term.

We first seek S in the form of a sum of binary functions:

$$S(\mathbf{r}_1,\ldots,\mathbf{r}_N)=\sum_{j_1 j_2} S(|\mathbf{r}_{j_1}-\mathbf{r}_{j_2}|). \tag{3.3}$$

We decompose the function $S(r)$ in a Fourier series

$$S(r)=\frac{1}{V}\sum_{\mathbf{k}}\sigma(k)e^{i(\mathbf{kr})}, \tag{3.4}$$

and write (3.3) in the form

$$S(\mathbf{r}_1,\ldots,\mathbf{r}_N)=\frac{N}{V}\sum_{\mathbf{k}}\sigma(k)\rho_{\mathbf{k}}\rho_{-\mathbf{k}}. \tag{3.5}$$

making use of Eq. (2.1). With the help of Eqs. (2.4) and (3.5), Eq. (3.2) becomes

$$\frac{N}{V}\sum_{\mathbf{k}}\left(\frac{\hbar^2k^2}{m}\sigma(k)+\frac{\nu(k)}{2}\right)\rho_{\mathbf{k}}\rho_{-\mathbf{k}} \tag{3.6}$$

$$+\frac{2}{N}\left(\frac{N}{V}\right)^2\sum_{\mathbf{k}_1\mathbf{k}_2;\,j}\sigma(k_1)\sigma(k_2)\frac{\hbar^2(\mathbf{k}_1\mathbf{k}_2)}{m}$$

$$\times\exp\{-i(\mathbf{k}_1+\mathbf{k}_2)\mathbf{r}_j\}\rho_{-\mathbf{k}_1}\rho_{-\mathbf{k}_2}=E-E_0,$$

where

$$E_0=\frac{1}{2}\frac{N^2}{V}\nu(0)-\frac{1}{2}\sum_{\mathbf{k}}\frac{N}{V}\nu(k) \tag{3.7}$$

$$+\frac{N}{V}\sum_{\mathbf{k}}\sigma(k)\frac{\hbar^2k^2}{m}.$$

An exact solution of Eq. (3.6) is difficult, but it is possible to simplify the procedure if we leave only the principle part (corresponding to $\mathbf{k}_1+\mathbf{k}_2=0$) in the second sum. This procedure is equivalent to a neglect of the correlation between particles. For the present, we accept this rough approximation, since it permits us to establish a simple connection between the method of "auxiliary variables" and the method of Bijl. Below, in Sec. 4, a more rigorous method is given for obtaining the wave function of the lowest state of a system of Bose particles.

After some simplification Eq. (3.6) takes the form

$$\frac{N}{V}\sum_{\mathbf{k}}\left\{-2\frac{N}{V}\frac{\hbar^2k^2}{m}\sigma^2(k)+\frac{\hbar^2k^2}{m}\sigma(k)+\frac{\nu(k)}{2}\right\} \tag{3.8}$$

$$\times\rho_{\mathbf{k}}\rho_{-\mathbf{k}}=E-E_0,$$

which can be satisfied by setting

$$-\frac{2N}{V}\frac{\hbar^2k^2}{m}\sigma^2(k)+\frac{\hbar^2k^2}{m}\sigma(k)+\frac{\nu(k)}{2}=0, \tag{3.9}$$

whence

$$\sigma(k)=\frac{1}{4n}-\frac{1}{4n}\frac{\sqrt{n\nu(k)(\hbar^2k^2/m)+(\hbar^2k^2/2m)^2}}{\hbar^2k^2/2m}, \tag{3.10}$$

$$n=\frac{N}{V}.$$

The negative sign is chosen for the radical since we are interested in the minimum value of the energy.

The logarithm of the wave function of the lowest state will be

$$S=\sum_{\mathbf{k}}\frac{1}{4} \tag{3.11}$$

$$\times\left\{1-\frac{\sqrt{n\nu(k)(h^2k^2/m)+(h^2k^2/2m)^2}}{h^2k^2/2m}\right\}\rho_{\mathbf{k}}\rho_{-\mathbf{k}}$$

and the corresponding wave function is

$$\rho_0=\exp\left\{\frac{1}{4}\sum_{\mathbf{k}}\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\right\} \tag{3.12}$$

$$\times\exp\left\{-\frac{1}{4}\sum_{\mathbf{k}}\frac{E(k)}{\hbar^2k^2/2m}\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\right\},$$

where $E(k)$ is the energy of elementary excitations in the weakly non-ideal Bose gas, given by Eq. (2.12).

The first factor in the wave function (3.12) is identical to the factor which we introduced in the wave function of the lowest state in order to make the principal part of the Hamiltonian (2.6) Hermitian. It can be demonstrated by direct substitution that the function (3.12) satisfies the principal part of Eq. (2.6).

If we neglect the term in Eq. (3.9) which is quadratic in $\sigma(k)$, we get Bijl's result:

$$\sigma(k) = -m\nu(k)/2\hbar^2k^2. \tag{3.13}$$

The logarithm of the wave function of the lowest state in this case is

$$S = \sum_{\mathbf{k}} -\frac{m\nu(k)n}{2\hbar^2k^2}\rho_{\mathbf{k}}\rho_{-\mathbf{k}} \tag{3.14}$$

and the corresponding wave function is

$$\varphi_0 = \exp\left\{-\frac{n}{4}\sum_{\mathbf{k}}\frac{\nu(k)}{\hbar^2k^2/2m}\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\right\}. \tag{3.15}$$

The function (3.16) satisfies the differential equation

$$\sum_{\mathbf{k}}\left\{\frac{\hbar^2k^2}{2m}\rho_{\mathbf{k}}\frac{\partial\varphi}{\partial\rho_{\mathbf{k}}} + \frac{1}{2}n\nu(k)\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\varphi\right\} = (E - E_0')\varphi, \tag{3.16}$$

where

$$E_0' = \frac{N^2}{2V}\nu(0) - \frac{1}{2}\sum_{\mathbf{k}} n\nu(k).$$

Equation (3.16) differs from the principal part of Eq. (2.8) in that part of the kinetic energy operator of the phonons is missing from it:

$$-\frac{\hbar^2k^2}{2m}\frac{\partial^2}{\partial\rho_{\mathbf{k}}\partial\rho_{-\mathbf{k}}}$$

This omission is unjustified, and, consequently, Bijl's method always gives a rough approximation to the wave function of lowest order of the system of Bose particles. Thus the doubt expressed by Landau and Lifshits[7] on the validity of one of the author's results[2], since they did not agree with those of Bijl[3], is unjustified.

Making use of (2.4) and (2.1), and carrying out the integration over the variable $\mathbf{k}$, we write the logarithm of the wave function in the form

$$S = -\frac{m}{2\hbar^2}\frac{1}{4\pi}\sum_{i,j}\int\frac{\Phi(r)\,d\mathbf{r}}{|\mathbf{r}_i - \mathbf{r}_j - \mathbf{r}|}, \tag{3.17}$$

or, after integration over the angular variables,

$$S = -\frac{m}{\hbar^2}\frac{1}{2}\sum_{i,j}\left\{\int_{r_{ij}}^{\infty}\Phi(r)\,r\,dr + \frac{1}{r_{ij}}\int_0^{r_{ij}}\Phi(r)\,r^2dr\right\}, \tag{3.18}$$

$$r_{ij} = |r_i - r_j|.$$

This form of the wave function coincides with that deduced by Bijl[3].

In obtaining Eq. (3.18) from Eq. (3.14), we have carried out a five-fold integration over the three variables $\mathbf{k}$ and the two angular variables of the radius vector $\mathbf{r}$. It also would have been possible to obtain Eq. (3.18) directly, by solving the Schrödinger equation in the variables $\mathbf{r}_j$, neglecting the quadratic terms in S, as was done by Bijl[3].

The equation (3.11) for the logarithm of the wave function can also be written in the form

$$S = -\frac{1}{4\pi^2}\frac{m}{\hbar^2 n} \times \sum_{i,j}\frac{1}{r_{ij}}\int_0^{\infty}\left\{E(k) - \frac{\hbar^2k^2}{2m}\right\}\frac{\sin kr_{ij}}{k}\,dk, \tag{3.19}$$

where the integration has been carried out over the angular variables.

4. THE WAVE FUNCTIONS OF THE LOWEST STATE OF A SYSTEM OF INTERACTING BOSE PARTICLES

In the preceding section, we detrmined the zeroth approximation wave function of the lowest state of a system of interacting Bose particles, making more precise the method of Bijl. We now return to the more systematic method of the Bose system, which makes possible the determination of the wave functions of the lowest state in the form of a decomposition in powers of a parameter of the smallness of interaction.

As a subsidiary problem, we determine the eigenfunctions of the operator

$$H_0 = \frac{1}{2}\sum_{\mathbf{k}} E(k)\left(-\frac{\partial^2}{\partial q_{\mathbf{k}}\partial q_{-\mathbf{k}}} + q_{\mathbf{k}}q_{-\mathbf{k}}\right), \tag{4.1}$$

which is the principal part of the Hamiltonian (2.11) The equation for the eigenfunctions of Eq. (4.1) can be written in the form

$$\frac{1}{2}\sum_{\mathbf{k}} E(k)\left\{-\frac{1}{4}\left(\frac{\partial^2}{(\partial q_{\mathbf{k}}^c)^2} + \frac{\partial^2}{(\partial q_{\mathbf{k}}^s)^2}\right) + (q_{\mathbf{k}}^c)^2 + (q_{\mathbf{k}}^s)^2\right\}\Phi = \mathcal{E}\Phi, \tag{4.2}$$

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[7] L. D. Landau and E. M. Lifshits, *Statistical Physics*, 2nd edition.

where $q^c_{\mathbf{k}}$ and $q^s_{\mathbf{k}}$ are the real and imaginary parts of $q_{\mathbf{k}}$. The eigenfunctions of Eq. (4.2) have the form

$$\Phi_{n_1,n_2} = \prod_{(\mathbf{k})}{}' \exp\{-q_{\mathbf{k}}q_{-\mathbf{k}}\} \tag{4.3}$$

$$\times\; H_{n_1}(\sqrt{2}\,q^c_{\mathbf{k}})H_{n_2}(\sqrt{2}\,q^s_{\mathbf{k}}).$$

Here n_1, n_2 are positive integers; H_n are the Hermitian polynomials; $\prod_{(\mathbf{k})}{}'$ denotes that the product omits the k which lie in the hemisphere (for example, $k_z > 0$), since $q^c_{\mathbf{k}}$ and $q^s_{\mathbf{k}}$ are connected by the relations $q^c_{\mathbf{k}} = q^c_{-\mathbf{k}}$ and $q_{\mathbf{k}}^s = -q^s_{-\mathbf{k}}$.

The function (4.3) corresponds to the energy

$$\mathscr{E} = \sum \frac{E(k)}{2}(1 + n_1 + n_2). \tag{4.4}$$

In the lowest energy state, $n_1 = n_2 = 0$ and the eigenfunction of (4.3) will be

$$\Phi_0 = \exp\left\{-\frac{1}{2}\sum_{\mathbf{k}} q_{\mathbf{k}}q_{-\mathbf{k}}\right\} \tag{4.5}$$

$$= \exp\left\{-\sum_{\mathbf{k}}\frac{\rho_{\mathbf{k}}\rho_{-\mathbf{k}}}{4\lambda^2_{\mathbf{k}}}\right\}.$$

By means of Eq. (2.7) we get for the wave function of the lowest state

$$\varphi_0 = \exp\left\{\frac{1}{4}\sum_{\mathbf{k}}\left(1 - \frac{1}{\lambda^2_{\mathbf{k}}}\right)\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\right\}, \tag{4.6}$$

which coincides with the function (3.12) obtained earlier, since we have, from Eqs. (2.10) and (2.12):

$$E(k) = \hbar^2k^2/\lambda^2_{\mathbf{k}}2m. \tag{4.7}$$

One can verify directly that the wave function (4.5) corresponds to the lowest energy level by demonstrating that the $b_{\mathbf{k}}$ --the operators of "annihilation" of phonons-- yield zero when they operate on (4.5)

The wave function of the state in which there is one phonon with wave number k is

$$\Phi_{\mathbf{k}} = \overset{*}{b}_{\mathbf{k}}\,\Phi_0 = \sqrt{2}\,q_{-\mathbf{k}}\Phi_0 \tag{4.8}$$

$$= \frac{1}{\lambda_{\mathbf{k}}}\frac{1}{\sqrt{N}}\sum_{j=1}^{N}\exp\{i(\mathbf{k}\mathbf{r}_j)\}\,\Phi_0.$$

In a similar way we can obtain the wave function of the state in which n phonons are present. We note that the $\rho_{\mathbf{k}}$ appear in the eigenfunctions of the ideal Bose gas, corresponding to the energy $\hbar^2k^2/2m$.

The wave functions (4.5), (4.6) and (4.8) of the lowest and excited states of a Bose gas are wave functions of zero approximation only, and are not of sufficient accuracy to permit us to find wave functions of arbitrary approximation. We shall now give an account of the method of constructing such functions for the lowest energy state of the system.

The Schrödinger equation for the system of interacting Bose particles in the $\rho_{\mathbf{k}}$ representation (2.14) can be written in the form

$$\{H_0 + H_1 + H_2\}\,\Phi = \mathscr{E}\Phi, \tag{4.9}$$

where

$$H_0 = \sum_{\mathbf{k}} E(k)\,b^+_{\mathbf{k}}b_{\mathbf{k}} + \frac{1}{\sqrt{N}} \tag{4.10}$$

$$\times \sum_{\substack{\mathbf{k}_1,\mathbf{k}_2\\ \mathbf{k}_1+\mathbf{k}_2\neq 0}} \frac{\hbar^2}{8m}(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}(b_{\mathbf{k}_1+\mathbf{k}_2} + b^+_{-\mathbf{k}_1-\mathbf{k}_2})$$

$$\times(1 + \lambda^2_{\mathbf{k}_1})(1 + \lambda^2_{\mathbf{k}_2})\,b_{-\mathbf{k}_1}b_{-\mathbf{k}_2};$$

$$H_1 = -\frac{1}{\sqrt{N}}$$

$$\times \sum_{\substack{\mathbf{k}_1,\mathbf{k}_2\\ \mathbf{k}_1+\mathbf{k}_2\neq 0}} \frac{\hbar^2}{8m}(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}(b_{\mathbf{k}_1+\mathbf{k}_2} + b^+_{-\mathbf{k}_1-\mathbf{k}_2})$$

$$\times\,[(1 + \lambda^2_{\mathbf{k}_1})(1 - \lambda^2_{\mathbf{k}_2})\,b^+_{\mathbf{k}_2}b_{-\mathbf{k}_1}$$

$$+ (1 + \lambda^2_{\mathbf{k}_2})(1 - \lambda^2_{\mathbf{k}_1})\,b^+_{\mathbf{k}_1}b_{-\mathbf{k}_2}],$$

$$H_2 = \frac{1}{\sqrt{N}}\sum\frac{\hbar^2}{8m}(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}$$

$$\times(b_{\mathbf{k}_1+\mathbf{k}_2} + b^+_{-\mathbf{k}_1-\mathbf{k}_2})(1 - \lambda^2_{\mathbf{k}_1})(1 - \lambda^2_{\mathbf{k}_2})\,b^+_{\mathbf{k}_1}b^+_{\mathbf{k}_2};$$

$$\mathscr{E} = E - E_0.$$

Here the Hamiltonian operator is divided into three parts: H_0, H_1, and H_2, corresponding to the zero , first, and second order relative to the small parameter

$$1 - \lambda^2_{\mathbf{k}} = 1 - \sqrt{\frac{h^2k^2/4m}{n\nu(k) + (h^2k^2/4m)}}. \tag{4.11}$$

In the zeroth approximation we get the equation

$$H_0\Phi = \mathscr{E}\Phi, \tag{4.12}$$

which has the "vaccum" solution given in Eq. (4.5), $\Phi = \Phi_0, \mathscr{E}_0 = 0$.

The second, non-ideal part of the operator H_0 yields zero in operating Φ_0, and does not change the wave function in zeroth approximation.

We attempt to solve the equation (4.9) with the aid of the theory of excitations. We seek a solution of Eq. (4.9) in the form

$$\Phi = \Phi_0 + \Phi_1 + \Phi_2 + \ldots; \qquad (4.13)$$
$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_1 + \mathcal{E}_2 + \ldots,$$

where $\Phi_1, \Phi_2, \ldots; \mathcal{E}_1, \mathcal{E}_2, \ldots$ are the corrections of the first and second order to the wave function and the energy of the lowest state. Substituting Eq. (4.13) and comparing the terms of corresponding order, we get the following equations for the functions Φ_1, Φ_2:

$$(H_0 - \mathcal{E}_0)\Phi_1 = (\mathcal{E}_1 - H_1)\Phi_0, \qquad (4.14)$$
$$(H_0 - \mathcal{E}_0)\Phi_2 = (\mathcal{E}_1 - H_1)\Phi_1 + (\mathcal{E}_2 - H_2)\Phi_0,$$

Inasmuch as

$$H_1\Phi_0 = 0,$$

the first equation of the set (4.14) is satisfied for $\Phi_1 = 0$, $\mathcal{E}_1 = 0$, the corrections of the first approximation become zero. The second equation of the set (4.14), which gives ,i.e., the corrections to the wave function and the energy of the lowest state can be written in the form

$$(H_0 - \mathcal{E}_0)\Phi_2 = \mathcal{E}_2\Phi_0 - \frac{1}{\sqrt{N}} \qquad (4.15)$$
$$\times \sum_{\substack{\mathbf{k}_1,\mathbf{k}_2 \\ \mathbf{k}_1+\mathbf{k}_2\neq 0}} \frac{\hbar^2}{8m}(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}(1-\lambda^2_{\mathbf{k}_1})$$
$$\times (1-\lambda^2_{\mathbf{k}_2})\, b^+_{-\mathbf{k}_1-\mathbf{k}_2} b^+_{\mathbf{k}_1} b^+_{\mathbf{k}_2}\Phi_0.$$

We seek the solution of this equation in the form of a decomposition in the operator $b^+_{\mathbf{k}}$

$$\Phi_2 = \sum_{\mathbf{k}} L(\mathbf{k})\, b^+_{\mathbf{k}}\Phi_0 \qquad (4.16)$$
$$+ \sum_{\mathbf{k}_1,\mathbf{k}_2} M(\mathbf{k}_1, \mathbf{k}_2)\, b^+_{\mathbf{k}_1} b^+_{\mathbf{k}_2}\Phi_0$$
$$+ \sum_{\substack{\mathbf{k}_1,\mathbf{k}_2 \\ \mathbf{k}_1+\mathbf{k}_2\neq 0}} N(\mathbf{k}_1, \mathbf{k}_2)\, b^+_{-\mathbf{k}_1-\mathbf{k}_2} b^+_{\mathbf{k}_1} b^+_{\mathbf{k}_2}\Phi_0,$$

where the functions $L(\mathrm{k})$, $M(\mathrm{k}_1, \mathrm{k}_2)$, $N(\mathrm{k}_1, \mathrm{k}_2)$ are to be determined.

Substituting (4.16) in Eqs. (4.15) and equating coefficients for terms which contain identical operators, we get the following set of equations for the unknown functions L, M, N and $\mathcal{E}_2$:

$$E(\mathbf{k})L(\mathbf{k}) = \frac{1}{\sqrt{N}}\sum_{\mathbf{k}'}\frac{\hbar^2}{4m}(\mathbf{k}', \mathbf{k}+\mathbf{k}') \qquad (4.17)$$
$$\times \frac{\lambda_{\mathbf{k}}}{\lambda_{\mathbf{k}'+\mathbf{k}}\lambda_{\mathbf{k}'}}(1+\lambda^2_{\mathbf{k}+\mathbf{k}'})(1+\lambda^2_{\mathbf{k}'}) \times M(\mathbf{k}+\mathbf{k}', -\mathbf{k}'),$$

$$\{E(\mathbf{k}_1) + E(\mathbf{k}_2)\}\, M(\mathbf{k}_1, \mathbf{k}_2)$$
$$= \delta^{-\mathbf{k}_2}_{\mathbf{k}_1}\frac{1}{\sqrt{N}}\sum_{\mathbf{k}'_2}\frac{\hbar^2}{4m}(\mathbf{k}'_2, \mathbf{k}_1 + \mathbf{k}'_2)\frac{\lambda_{\mathbf{k}_1}}{\lambda_{\mathbf{k}_1+\mathbf{k}'_2}\lambda_{\mathbf{k}'_2}}$$
$$\times (1+\lambda^2_{\mathbf{k}_1+\mathbf{k}'_2})(1+\lambda^2_{\mathbf{k}'_2})\{N(-\mathbf{k}_1-\mathbf{k}'_2, -\mathbf{k}'_2)$$
$$+ N(\mathbf{k}_1, \mathbf{k}'_2) + N(\mathbf{k}'_2, \mathbf{k}_1)\},$$

$$\{E(\mathbf{k}_1+\mathbf{k}_2) + E(\mathbf{k}_1) + E(\mathbf{k}_2)\}\, N(\mathbf{k}_1, \mathbf{k}_2) \qquad (4.18)$$
$$= -\frac{1}{\sqrt{N}}\frac{\hbar^2}{8m}(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}(1-\lambda^2_{\mathbf{k}_1})(1-\lambda^2_{\mathbf{k}_2});$$
$$\mathcal{E}_2 = \frac{1}{\sqrt{N}}\sum_{\substack{\mathbf{k}_1,\mathbf{k}_2 \\ \mathbf{k}_1+\mathbf{k}_2\neq 0}}\frac{\hbar^2}{4m}(\mathbf{k}_1\mathbf{k}_2)(1+\lambda^2_{\mathbf{k}_1})$$
$$\times (1+\lambda^2_{\mathbf{k}_2})\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}}\{N(-\mathbf{k}_1, -\mathbf{k}_2) +$$
$$+ N(\mathbf{k}_1+\mathbf{k}_2, -\mathbf{k}_2) + N(-\mathbf{k}_1, \mathbf{k}_1+\mathbf{k}_2)\}.$$

We see from the second equation of (4.17) that the function $M(\mathrm{k}+\mathrm{k}', -\mathrm{k}')$ is different from zero only for $\mathrm{k} = 0$, which is impossible, since $\mathrm{k} \neq 0$, by assumption; consequently $L(k) = 0$.

We find the value of $\mathcal{E}_2$ from Eq. (4.18):

$$\mathcal{E}_2 = -\frac{2}{N} \qquad (4.19)$$
$$\times \sum_{\substack{\mathbf{k}_1,\mathbf{k}_2 \\ \mathbf{k}_1+\mathbf{k}_2\neq 0}}\left(\frac{\hbar^2}{8m}\right)^2\frac{(\mathbf{k}_1\mathbf{k}_2)(1+\lambda^2_{\mathbf{k}_1})(1+\lambda^2_{\mathbf{k}_2})\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}\{E(\mathbf{k}_1+\mathbf{k}_2)+E(\mathbf{k}_1)+E(\mathbf{k}_2)\}}$$
$$\times \left\{(\mathbf{k}_1\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1+\mathbf{k}_2}}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}} \times (1-\lambda^2_{\mathbf{k}_1})(1-\lambda^2_{\mathbf{k}_2})\right.$$
$$- (\mathbf{k}_2, \mathbf{k}_1+\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_1}}{\lambda_{\mathbf{k}_1+\mathbf{k}_2}\lambda_{\mathbf{k}_2}}(1-\lambda^2_{\mathbf{k}_1+\mathbf{k}_2})(1-\lambda^2_{\mathbf{k}_2})$$
$$\left. - (\mathbf{k}_1, \mathbf{k}_1+\mathbf{k}_2)\frac{\lambda_{\mathbf{k}_2}}{\lambda_{\mathbf{k}_1+\mathbf{k}_2}\lambda_{\mathbf{k}_1}}(1-\lambda^2_{\mathbf{k}_1+\mathbf{k}_2})(1-\lambda^2_{\mathbf{k}_1})\right\}$$

Because of the practical continuity of the spectrum, we can go from the summation to integration in Eq. (4.19) and obtain an expression for $\mathcal{E}_2$ in the form

$$\mathcal{E}_2 = -\frac{2N}{(2\pi)^6 n^2}\left(\frac{\hbar^2}{8m}\right)^2 \qquad (4.20)$$
$$\times \iint \frac{(\mathbf{k}_1\mathbf{k}_2)\lambda^2_{\mathbf{k}_1+\mathbf{k}_2}(1-\lambda^4_{\mathbf{k}_1})(1-\lambda^4_{\mathbf{k}_2})d\mathbf{k}_1 d\mathbf{k}_2}{\lambda^2_{\mathbf{k}_1}\lambda^2_{\mathbf{k}_2}\{E(\mathbf{k}_1+\mathbf{k}_2)+E(\mathbf{k}_1)+E(\mathbf{k}_2)\}}$$

$$+\frac{4N}{(2\pi)^6 n^2}\left(\frac{\hbar^2}{8m}\right)^2$$

$$\iint \frac{(\mathbf{k}_1,\, \mathbf{k}_1+\mathbf{k}_2)(\mathbf{k}_1\mathbf{k}_2)(1-\lambda^4_{\mathbf{k}_1})(1-\lambda^2_{\mathbf{k}_1+\mathbf{k}_2})(1+\lambda^2_{\mathbf{k}_2})\, d\mathbf{k}_1 d\mathbf{k}_2}{\lambda^2_{\mathbf{k}_1}\{E(\mathbf{k}_1+\mathbf{k}_2)+E(\mathbf{k}_1)+E(\mathbf{k}_2)\}}\,.$$

For second order corrections to the wave function of the lowest state, we get

$$\Phi_2 = \frac{-2}{N}\sum_{\mathbf{k}_1,\mathbf{k}_2}\left(\frac{\hbar^2}{8m}\right)^2 \qquad (4.21)$$

$$\frac{(\mathbf{k}_1\mathbf{k}_2)(\mathbf{k}_2,\, \mathbf{k}_1+\mathbf{k}_2)(1+\lambda^2_{\mathbf{k}_1+\mathbf{k}_2})(1-\lambda^2_{\mathbf{k}_1})(1-\lambda^4_{\mathbf{k}_2})\, b^+_{\mathbf{k}_1} b^+_{-\mathbf{k}_1}\Phi_0}{\lambda^2_{\mathbf{k}_2} E(\mathbf{k}_1)\{E(\mathbf{k}_1+\mathbf{k}_2)+E(\mathbf{k}_1)+E(\mathbf{k}_2)\}}$$

$$-\frac{1}{\sqrt{N}}$$

$$\sum_{\substack{\mathbf{k}_1,\mathbf{k}_2\\ \mathbf{k}_1+\mathbf{k}_2\neq 0}} \frac{\hbar^2}{8m}\, \frac{(\mathbf{k}_1\mathbf{k}_2)\lambda_{\mathbf{k}_1+\mathbf{k}_2}(1-\lambda^2_{\mathbf{k}_1})(1-\lambda^2_{\mathbf{k}_2})\, b^+_{-\mathbf{k}_1-\mathbf{k}_2} b^+_{\mathbf{k}_1} b^+_{\mathbf{k}_2}\Phi_0}{\lambda_{\mathbf{k}_1}\lambda_{\mathbf{k}_2}\{E(\mathbf{k}_1+\mathbf{k}_2)+E(\mathbf{k}_1)+E(\mathbf{k}_2)\}}$$

$$+\frac{1}{N}\sum_{\mathbf{k}_1,\mathbf{k}_2}\left(\frac{\hbar^2}{8m}\right)^2$$

$$\times\frac{(\mathbf{k}_2,\mathbf{k}_1+\mathbf{k}_2)^2\lambda^2_{\mathbf{k}_1}(1-\lambda^4_{\mathbf{k}_1+\mathbf{k}_2})(1-\lambda^4_{\mathbf{k}_2})\, b^+_{\mathbf{k}_1} b^+_{-\mathbf{k}_1}\Phi_0}{\lambda^2_{\mathbf{k}_1+\mathbf{k}_2}\lambda^2_{\mathbf{k}_2} E(\mathbf{k}_1)\{E(\mathbf{k}_1+\mathbf{k}_2)+E(\mathbf{k}_1)+E(\mathbf{k}_2)\}}.$$

With the aid of Eqs. (4.5), (2.13), (2.9) and (2.1) we can represent (4.21) in the form of an explicit function of coordinates.

By a suitable method we can determine Φ_3 and higher terms of the decomposition of the wave function of the lowest state of the Bose gas in powers of the parameter of the smallness of the interaction.

The method of the "auxiliary variables" developed above for application to the Bose gas has a simple relation to the method of approximate second quantization of the weakly non-ideal Bose gas[2].

We express $\rho_{\mathbf{k}}$ (2.1) in the representation of second quantization in momentum space:

$$\rho_{\mathbf{k}} = \frac{1}{\sqrt{N}}\sum_{\mathbf{f}} a^+_{\mathbf{f}-\mathbf{k}} a_{\mathbf{f}} \quad (\mathbf{k}\neq 0), \qquad (4.22)$$

where $a^+_{\mathbf{f}}$, $a_{\mathbf{f}}$ are Bose operators. We separate in Eq. (4.22) terms which contain Bose operators with zero momentum and write $\rho_{\mathbf{k}}$ in the form

$$\rho_{\mathbf{k}} = \frac{a^+_{-\mathbf{k}} a_0}{\sqrt{N}} + \frac{a^+_0 a_{\mathbf{k}}}{\sqrt{N}} \qquad (4.23)$$

$$+\frac{1}{\sqrt{N}}\sum_{\substack{\mathbf{f}\\ (\mathbf{f}\neq 0,\ \mathbf{f}\neq\mathbf{k})}} a^+_{\mathbf{f}-\mathbf{k}} a_{\mathbf{f}}.$$

The first two terms in (4.23) represent the Bose operators $b_{\mathbf{k}}$ and $b^+_{\mathbf{k}}$ which were used in our previous work[2].

We note that the wave function of the elementary excitations of the zeroth approximation (4.8) is identical in form with the function hypothesized by Feynman in his work on the theory of superfluidity of He II[8]. The correlation function of the zeroth approximation, which can be calculated with the help of the variation derivative of the energy of the lowest state (2.15) with respect to the function of interaction by the relation

$$g(r) = \frac{2V}{N^2}\,\frac{\delta E_0}{\delta\Phi(r)}, \qquad (4.24)$$

is also identical to the correlation function calculated by Feynman. Thus we see that the results of Feynman agree with the zeroth approximation of our method. The further development of the methods of the present work can serve as the basis for improving the theory of superfluidity of He II.

Translated by R. T. Beyer
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[8] R. P. Feynman, Phys. Rev. 94, 262 (1954)

ERRATA

	reads	should read
Rodionov et al, Soviet Phys. JETP 1, 64 (July, 1955)		
p. 64, column 2, line 19	methyl or ethyl	methylal or ethylal
Bogoliubov and Zubarev, Soviet Phys. 1, 83 (July, 1955)		
p. 88, column 1, line 6	omits	is taken over
p. 88, col. 1, 4th line from bottom	appears in	are
Grametitskii et al, Soviet Phys. JETP 1, 562 (November, 1955)		
p. 562, title	"fissions"	"disintegrations"
p. 562, paragraphs 1 and 2 (3times)	"fissions"	"disintegrations"
p. 563, column 1, 5th line from bottom	"fissions"	"disintegrations"
Index to J. Tech. Phys.		
p. 374		

Our apologies to Academician Abram Fedorovich Ioffe for interpreting his anniversary biography as an obituary. Like the report of Mark Twain's demise, it was a great exaggeration.

ANNOUNCEMENT

Beginning with the next issue (August, 1956), Soviet Physics JETP will appear monthly. Volume 3 (August, 1956-January, 1957) will contain translations of all articles appearing in Volume 30 of the Journal of Experimental and Theoretical Physics of the USSR (January-June, 1956). Volume 4 (February-July, 1957) will be a translation of Volume 31 (July-December, 1956) of the Soviet Journal.

Subscribers to Soviet Physics JETP for the calender year 1956 will receive Volume 3 as the completion of their subscription. For new subscribers (beginning with Volume 3) the subscription price will be $60.00 per year (two volumes, twelve issues) in the United States and Canada, $64.00 per year elsewhere. The single issue price will remain at $6.00.

Beginning in either July or August, 1956, the American Institute of Physics will commence translations of the Journal of Technical Physics of the USSR, the Acoustics Journal of the USSR and the physical sciences portions of Doklady. Price schedules for these journals are given below. Subscriptions should be addressed to the American Institute of Physics, 57 East 55 Street, New York 22, N. Y.

Soviet Physics—Technical Physics

A translation of the "Journal of Technical Physics" of the Academy of Sciences of the U.S.S.R. Eighteen issues per year, approximately 4,000 Russian pages. Annually, $90.00 domestic, $95.00 foreign.

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A translation of the "Journal of Acoustics" of the Academy of Sciences of the U.S.S.R. Four issues per year, approximately 500 Russian pages. Annually, $20.00 domestic, $22.00 foreign. The 1955 issues of "Journal of Acoustics" U.S.S.R. will also be published. Will consist of two volumes, approximately 500 pages, and the subscription price will be $20.00 for the set.

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